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Development of Proxy Models for Predicting and Optimizing the Time and Recovery Factor at Breakthrough During Water Injection in Oil Reservoirs

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Abstract: Numerical reservoir simulation studies can be used to plan water injection projects to delay time and maximize oil recovery at water breakthrough which is time-consuming and computationally expensive. Combining computationally inexpensive proxy models and optimization algorithms is a solution to this problem. In this study, the Box-Behnken design method and response surface methodology were used to develop two proxy models which showed the relationship between time and recovery factor at water breakthrough with six independent variables namely porosity, horizontal permeability, water viscosity, bottom-hole pressure, water injection rate and vertical permeability. A comparison of actual and predicted values for time and oil recovery factor at water breakthrough was found to be in good agreement with each other. An average absolute percentage error of 2.038% and 1.217%, a root mean square error of 0.08 and 0.0000988, and coefficients of determination, R^2 of 0.9984 and 0.9946 were obtained for time and recovery factor at water breakthrough respectively. These are indications that the developed models are accurate, valid, and reliable. The models were further validated by comparing the actual and predicted water breakthrough time and recovery factor at water breakthrough using input variables that were not used in model development. These were also in close agreement with each other. The MATLAB multi-objective genetic algorithm was used to determine at a specific average porosity and permeability value, the best optimum controllable variables that maximized the objective functions. These were found to be 10.8978 years and 0.786 respectively and agreed with simulation results obtained using similar input parameter values.

Keywords: Reservoir Simulation, Proxy Model, Design of Experiments, Breakthrough Time, Recovery Factor, Optimization

1. Introduction

1.1. Background of the Study

A typical petroleum reservoir usually undergoes primary, secondary, and enhanced oil recovery processes during its lifetime [1]. Ahmed & Meehan reported that Primary, Secondary, and Tertiary recovery processes will result respectively to a 25%, 30%, and 45% of Original Oil in Place for light oils, and 5%, 5%, and 90% of original oil in place for heavy oils respectively [2]. Primary oil recovery methods consist of various reservoir drive mechanisms such as gas cap expansion, solution gas drive, rock expansion, water drive, and gravity drainage and are defined as a distinct form of energy within a reservoir that causes expulsion or production of reservoir fluids to the surface [3]. These methods result in the initial production stage of a reservoir. Secondary oil recovery methods are usually implemented to drive more oil from the reservoir towards the production wells when primary oil recovery processes are no longer feasible. These processes are used to increase hydrocarbon recovery beyond primary recovery [4]. Secondary oil recovery involves injecting water or gas to provide pressure maintenance and sweep crude oil towards the production wells. An immiscible gas injection process is not as effective as a waterflooding process, causing secondary oil recovery to be used almost synonymously for waterflooding [5]. Enhanced Oil Recovery (EOR) processes can be classified as Miscible Gas Injection, Chemical Processes, and Thermal Processes. Fluids injected into a reservoir for EOR purposes supplement the natural energy of the reservoir to displace oil to the production wells, interact with the rock-oil system to create favorable conditions for oil recovery by lowering interfacial tension, oil swelling, reduction in oil viscosity, and wettability modifications [5].

An equation for predicting water breakthrough time and critical rate for an Iranian oil field was developed by Karami et al. [6]. The water coning correlation developed by the authors can be used to predict the critical rate and water breakthrough time which was found to be useful in determining the optimum rate to delay water breakthrough time and hence know when water coning reaches the production well. A novel model for predicting water breakthrough time in a high sulfur gas reservoir with edge water was developed by Guo et al. [7]. The model was developed based on the flow law and sulfur precipitation model in porous media. In this model, water breakthrough time was dependent on irreducible water saturation, residual gas saturation, and the distance between the gas wells and the edge water, sulfur saturation, and non-Darcy flow. This model and existing models were in good agreement showing the validity and accuracy of the proposed model. Ignoring the effect of condensate oil during water breakthrough time calculations can cause a difference between actual and predicted results. This issue was addressed by Huang et al. [8] when they developed a new model for predicting water breakthrough time for gas condensate reservoir under edge water drive. Their model considered a variation of condensate saturation with time and the distance from the bottom of the well, which is in line with the actual situation of change in condensate saturation in the formation. The authors concluded that the main factors affecting water breakthrough time in gas condensate reservoirs are daily gas production, gas viscosity, porosity, permeability, gas-water mobility ratio, gas layer thickness, and the distance between the initial gas-water boundary and the bottom of the well.

Water injection is normally conducted as a secondary recovery method after primary oil recovery processes have been implemented [9]. This is because of the low cost of water, and water properties which enhance the sweep of oil towards the production well [10]. During water injection in petroleum reservoirs, there is a tendency for the injected water to breakthrough at the production wells after a period of water injection, which results in high water cuts which continues until it becomes uneconomical to run the field [11]. This is referred to as water breakthrough time and it is a function of well locations, controls placed on wells, and reservoir heterogeneity. The factors which affect waterflooding were reported by Latuan et al. [12] to include reservoir geometry, reservoir fluid characteristics, reservoir depth, reservoir heterogeneity and continuity, fluid saturations, lithology, and fluid characteristics, and reservoir drive mechanism. The authors also highlighted that reservoir heterogeneity, high permeability zone, poor cement bonding, and unfavorable mobility ratios to be the causes of early water breakthrough during water injection.

It is desirable to delay the time and maximize recovery at water breakthrough during water injection because it mitigates low oil productivity, minimizes corrosion of surface and downhole equipment, and solves water disposal challenges [13].

A methodology was presented by Meshioye et al. [11] in which smart injector well technology was used to control waterflooding aimed at maximizing net present value. Inflow control valves were installed on the smart injector wells which could open or close automatically to meet certain production requirements. Reservoir simulation techniques were used in a case study to optimize waterflooding by Ogbeiwi et al. [14]. The authors used a simple optimization methodology in which the effects of zones of production and injection, waterflood pattern, and number/type of producers and injectors on cumulative oil recovery were analyzed. Results from their study showed that water injection in more zones resulted in effective pressure maintenance, higher water production was observed from the use of vertical injectors, and higher cumulative production was achieved with horizontal injectors if water is injected in the zone from where oil is produced.

1.2. Problem Statement

Simulating water injection in a given reservoir before implementation has been an approach used by numerous researchers to determine the optimum parameters that can delay the time and maximize recovery factor at water breakthrough. Variables such as water injection rate, the viscosity of injected water, and well bottom-hole pressure for production wells are control parameters that need to be considered when simulating water injection processes in petroleum reservoirs. Conducting numerous reservoir simulation studies can be used to determine the set of parameters that will delay water breakthrough time and maximize oil recovery at breakthrough. However, determining an optimum set of parameters that will meet these objectives is a challenge as an infinite number of reservoir simulation runs need to be conducted to arrive at an optimum solution. This approach is computationally expensive and time-consuming because it involves entering specific parameters into the reservoir simulation model and running simulations to determine an output. This process is continued till a maximum output is generated by a set of parameters considered optimal. A solution to this problem is to develop reservoir simulation proxy models using statistical or data-driven methods which have proven to consume less time and are computationally inexpensive, and the parameters which can maximize or minimize an objective function as desired can be determined with minimal computational effort using optimization algorithms.

2. Literature Review

Da Silva et al. highlighted the advantage of using proxy models in reservoir simulation studies especially when it is impossible to directly evaluate a system or when the process is computationally expensive [15]. Proxy models can be polynomial regression models, ordinary kriging models, artificial neural network models, response surface methodology, and the design of experiments [15].

Yasutra et al. developed a proxy model for predicting waterflood performance in deltaic channeling sands with a normal five-spot well pattern [16]. The authors observed from their study that the developed proxy model was able, faster, reliable, and easy to use in predicting the waterflooding performance for these types of reservoirs. Olanipekun et al. used ~~a~~ design of experiments and response surface methodology approach in developing proxy models to aid the screening of candidate reservoirs for waterflooding and gas flooding [17]. The developed models were used to predict oil recovery factors under waterflood and gas flood conditions respectively and the model which results in a higher recovery would indicate the preferred injection method. Results from their study showed that the developed proxy models were robust and could be used as an initial screening study for water and gas flood candidates.

Research has also shown that the results from a proxy model does not necessarily give a 100% match with results from the numerical simulation model but a minimal range of error between the results can be obtained [18]. Proxy models can serve as an alternative to the reservoir simulation models and can aid in overcoming the challenges observed by running numerous reservoir simulations to arrive at an optimum solution [19]. The authors reported applications of proxy modeling in various aspects of petroleum engineering such as sensitivity analysis, assisted history matching, field development planning, risk analysis, optimization, and reservoir characterization. Group Method Data Handling (GDMH) type Neural Network was used by Daghbandan & Chalik in developing a proxy model for predicting Field Oil Production Rate (FOPR) in a reservoir during immiscible gas injection [20]. A higher degree of accuracy was observed when predicted performance was compared with the simulated performance of the reservoir. A proxy-based optimization approach was used by Al-Mudhafar & Rao to optimize future oil recovery during the Gas-Assisted Gravity Drainage process after conducting an acceptable history matching [21].

Most researchers have focused on modeling water breakthrough time during the primary phase of oil production, and especially for reservoirs with a bottom or edge water drive. Others have aimed at optimizing waterflood performance to maximize cumulative recovery. This study focuses on modeling the time and recovery factor at breakthrough during water injection in oil reservoirs using statistical proxies. Box-Behnken design of experiment method and response surface methodology was used to develop proxy models for predicting the time and recovery factor at breakthrough during water injection in oil reservoirs. The proxy models showed the relationship between the time and recovery factor at water breakthrough with porosity, vertical permeability, horizontal permeability, viscosity and rate of injected water, and bottom-hole pressure of the production well. With a suitable optimization algorithm (genetic algorithm), the search for an optimum set of parameters (water injection rate, viscosity of injected water, and bottom-hole pressure of production well) that will delay water breakthrough time and maximize oil recovery factor at breakthrough for a given reservoir of known average porosity and average permeability can be determined.

3. Materials and Method

3.1. Materials

The materials or tools used in this study are Eclipse100 reservoir simulation software, Design Expert 12 software, and Genetic Algorithm in MATLAB.

a. Eclipse100 reservoir was used to build the reservoir model and run the simulations to determine the time and recovery factor at water breakthrough for each simulation run.
b. Design Expert 12 software consists of the Box-Behnken design method which was used to generate parameter realizations which are the various input datasets for conducting simulation runs with Eclipse 100 reservoir simulator to arrive at desired responses. The corresponding results were used for analysis and equally a regression model was built using Design-Expert software.
c. Genetic Algorithm in MATLAB was used in optimizing the developed proxy model and aimed at determining the optimum set of parameters that will delay water breakthrough time and maximize oil recovery factor at breakthrough.

3.2. Method

3.2.1. Reservoir Description

The reservoir model used in this study is a single-phase model with grid dimensions of 10*10*3 in the x, y, and z directions respectively. The model is represented by 300 cells and each cell has a length of 150ft in the x and y direction and 50ft in the z-direction. The reservoir has a total length and width of 1500ft and a thickness of 150ft. Table 1 shows the reservoir dimensions, rock and fluid properties, and reservoir initialization data. Figure 1 shows an illustration of the developed static reservoir model used in this study.

The porosity and permeability of the reservoir model were used as input variables together with water injection rate, bottom-hole flowing pressure of the injection well, and viscosity of the injected water in the development of the proxy model. The injection and production wells were placed on grid (1, 1) and grid (10, 10) respectively and both wells were perforated on all three layers.

Table 1. *Properties of Reservoir model.*

Reservoir Property	Value
Number of grids	300 (10 x 10 x 3)
Reservoir thickness, (ft)	150
Reservoir length, (ft) x Reservoir Width, (ft)	1500 x 1500
Residual oil saturation	0.176
Initial oil saturation	O.843
Initial water saturation	0.157
Oil viscosity, μ (cp)	1.2
Initial oil formation volume factor, β_o (RB/STB)	1.2356
Oil density, ρ_o (Ib/ft^3)	49.1
Water density, ρ_w (Ib/ft^3)	64.79
Gas density, ρ_g (Ib/ft^3)	0.06054
Oil gravity, $°API$	54.7
Water formation volume factor, β_w (RB/STB)	1.03382
Bubble point pressure, P_b (psi)	3337
Rock compressibility, (psi^{-1})	2 * 10^{-5}
Rock pressure (psi)	329
Initial reservoir pressure at datum depth (psi)	4000

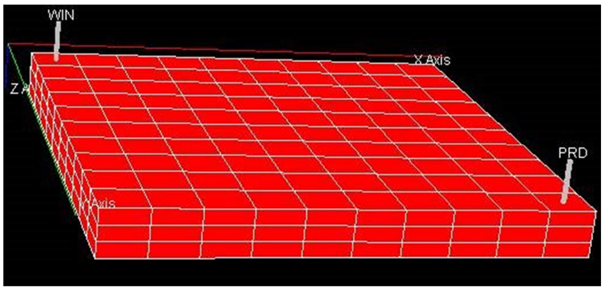

Figure 1. *Static Reservoir Model.*

3.2.2. Generating Input Data with Box-Behnken Design Method for Conducting Reservoir Simulation Runs

Box-Behnken design is a design method for fitting quadratic models requiring 3 levels of each factor. In this design, the treatment combinations are the midpoints of edges of the process space and at the center. These designs have fewer treatment combinations. The advantage of the Box-Behnken design method is that it requires fewer experimental runs, is most effective, is used to optimize the main, interaction, and quadratic effects, and requires a minimum of 3 factors and 3 levels. The number of experiments that can be generated from the Box-Behnken design method is given by equation 1

$$N = 2k * (k - 1) + C_o \quad (1)$$

Where N is the number of simulations, k is the number of factors, and C_o is the number of center points.

Response surface methodology is a useful tool for finding the relationship between a response and uncertain variables. This method combines mathematics and statistics in developing a simple empirical model of the system which can be easily optimized with an optimization algorithm.

The form of the relationship between the response and the independent variables might not be known at the initial stage but a suitable relationship can be found by starting with a linear or second-order polynomial of the independent variables. Equation 2 shows the typical response and independent variable relationship for a second-order polynomial function.

$$Y = \beta_o + \sum \beta_i X_i + \sum \beta_i X_{ii}^2 + \sum\sum \beta_{ij} X_{ij} \quad (2)$$

The coefficients $\beta_o, \beta_1, \beta_2 \ldots.. \beta_{k,} \beta_{ij}$ are referred to as regression coefficients of the response surface model and can be determined with regression techniques such as least square techniques. A model with several regression coefficients might not be as accurate as that with fewer coefficients and that with fewer coefficients can be improved by adding other coefficients to the model. As stated earlier, it is important to determine the most influential factors of a regression model.

A homogeneous three-dimensional reservoir model was developed using the data presented in Table 1 and Eclipse 100 Reservoir Simulator. Six independent variables namely porosity (X1), horizontal permeability (X2), water viscosity (X3), bottom-hole pressure (X4), water injection rate (X5), and vertical permeability (X6) were used in the development of the proxy model. The independent variables were fixed within specific ranges (minimum and maximum values) shown in Table 2 for generating sets of data for conducting simulation runs.

Table 2. *Range of Variables for Generating Simulation Runs.*

Name	Symbol	Units	Minimum	Maximum
Porosity	X1	Fraction	0.2000	0.6000
Horizontal Permeability	X2	Md	200.00	3230.00
Viscosity of Injected Water	X3	Cp	1.0000	10.00
Bottom hole pressure (BHP)	X4	Psi	3400.00	3700.00
Injection rate	X5	stb/day	2000.00	10000.00
Vertical Permeability	X6	Md	75.00	2435.00

3.2.3. Model Development

Using the data ranges specified in Table 2 and the Box-Behnken design method, datasets for conducting reservoir simulations were generated. Box-Behnken designs are used to generate higher-order response surfaces using fewer runs than a normal factorial technique. For six variables, 48 sets of data were generated using design expert 12 as shown in Table 3 for conducting simulation runs.

Table 3. *Simulation Runs for sets of input Variables.*

Run	Porosity (X1), fraction	Horizontal Permeability (X2), md	water viscosity (X3), cp	Bottom hole pressure, (X4), psi	Water Injection rate (X5), stb/day	Vertical Permeability (X6), md
1	0.4	3230	5.5	3550	2000	2435
2	0.6	1715	1	3550	6000	75
3	0.4	1715	10	3700	6000	2435
4	0.4	200	5.5	3550	2000	75
5	0.6	1715	1	3550	6000	2435
6	0.4	1715	10	3400	6000	75
7	0.6	1715	5.5	3700	2000	1255

Run	Porosity (X1), fraction	Horizontal Permeability (X2), md	water viscosity (X3), cp	Bottom hole pressure, (X4), psi	Water Injection rate (X5), stb/day	Vertical Permeability (X6), md
8	0.2	200	5.5	3700	6000	1255
9	0.2	1715	10	3550	6000	2435
10	0.2	3230	5.5	3400	6000	1255
11	0.2	1715	1	3550	6000	75
12	0.4	1715	10	3700	6000	75
13	0.2	1715	5.5	3700	2000	1255
14	0.4	3230	1	3550	2000	1255
15	0.6	1715	5.5	3700	10000	1255
16	0.2	1715	1	3550	6000	2435
17	0.4	200	10	3550	10000	1255
18	0.4	200	10	3550	2000	1255
19	0.2	1715	10	3550	6000	75
20	0.6	1715	5.5	3400	10000	1255
21	0.4	1715	1	3700	6000	2435
22	0.4	200	5.5	3550	2000	2435
23	0.4	3230	5.5	3550	10000	75
24	0.2	1715	5.5	3400	10000	1255
25	0.4	1715	1	3400	6000	2435
26	0.2	1715	5.5	3400	2000	1255
27	0.4	200	5.5	3550	10000	2435
28	0.6	1715	5.5	3400	2000	1255
29	0.6	3230	5.5	3400	6000	1255
30	0.4	3230	10	3550	2000	1255
31	0.2	3230	5.5	3700	6000	1255
32	0.4	1715	1	3400	6000	75
33	0.4	200	1	3550	10000	1255
34	0.2	1715	5.5	3700	10000	1255
35	0.4	200	5.5	3550	10000	75
36	0.2	200	5.5	3400	6000	1255
37	0.6	3230	5.5	3700	6000	1255
38	0.4	3230	10	3550	10000	1255
39	0.6	200	5.5	3400	6000	1255
40	0.6	1715	10	3550	6000	2435
41	0.4	3230	1	3550	10000	1255
42	0.4	1715	10	3400	6000	2435
43	0.4	200	1	3550	2000	1255
44	0.4	3230	5.5	3550	10000	2435
45	0.6	200	5.5	3700	6000	1255
46	0.4	3230	5.5	3550	2000	75
47	0.4	1715	1	3700	6000	75
48	0.6	1715	10	3550	6000	75

3.3. Running Reservoir Simulations

Each dataset presented in Table 3 was entered into the base reservoir model data file and a simulation conducted with Eclipse 100 reservoir simulator to determine the time and recovery factor at water breakthrough. The input and output data were entered into design expert 12 for further analysis.

3.4. Analysis of Variance (ANOVA)

Various regression models were evaluated and analyzed using analysis of variance (ANOVA) to ascertain the model which best fits input and output data. Various transformation functions are available in design expert, and each transformation function was evaluated as follows

a. The recommended transformation function should yield the highest value of the coefficient of determination R^2 value.
b. The transformation function which yields the lowest difference between Adjusted R^2 and Predicted R^2 which should be less than 0.2 is recommended.
c. The transformation function which yields the highest value of Adequacy in Precision (Adeq Precision) is recommended. Adeq Precision measures the signal-to-noise ratio, a value greater than 4 is desirable, and the greater the value indicates that the model can be used to navigate the design space.
d. Comparing the standard deviation obtained from different transformation functions. The transformation function which yields the least value of standard deviation is recommended.

The recommended transformation function for a set of input and output data can also be determined using the Box-Cox plot for power-law transformations. After the transformation function has been selected based on the stated criteria, an equation or proxy model for predicting the time and recovery factor at water breakthrough was developed from the recommended transformation function. The model was validated to determine the performance of the model within the experimental design space.

3.5. Model Validation

The developed proxy models were validated by comparing simulated and predicted results for each input dataset to see if they agree with each other. Also, a valid model should have a relatively high value of the coefficient of determination given by equation 3. Evaluation of the performance and validity of the models were also determined using the average absolute percentage error (AAPE) and root mean squared error (RMSE) which are given by equations 4 and 5 respectively.

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(Y_{sim} - Y_{pred}\right)^2}{\sum_{i=1}^{N}\left(Y_{pred} - \overline{Y_{sim}}\right)^2} \tag{3}$$

$$AAPE = \frac{1}{N}\sum_{i=1}^{N}\left(\sqrt{\left(\left(\frac{Y_{sim} - Y_{pred}}{Y_{sim}}\right)^2\right)}\right) * 100\% \tag{4}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_{sim} - Y_{pred}\right)^2}{n}} \tag{5}$$

3.6. Optimization of Proxy Model

Optimization algorithms have been used by different researchers in science and engineering to optimize engineering and industrial systems. Optimization can be single or multi-objective depending on the number of objective functions being considered. Single objective optimization involves the determination of an optimum solution of a model with a single objective function. Multi-objective optimization involves determining more than one optimal solution of more than one model with more than one objective function. In multi-objective optimization, different solutions might not satisfy the objective functions and this causes different solutions to produce tradeoffs between the different objectives. A set of solutions represented in a Pareto front will be generated.

In this study, the developed proxy models were coded in MATLAB and using the Global Optimization toolbox in MATLAB which contains the multi-objective genetic algorithm (MOGA), the sets of parameters that will maximize the objective functions were determined.

The general formulation of a multi-objective optimization problem is as follows

$$minimize/maximize\ f_n(x), n = 1, 2, 3, \ldots\ldots\ldots\ldots, n$$

Subject to $LB \le X(1) \le UB$

$$LB \le X(1) \le UB$$

$$LB \le X(3) \le UB$$

$$LB \le X(n) \le UB$$

Where $f_n(x)$ are n number of objective functions with respect to X(1), X(2), X(3),……, X(n)

LB is the lower bound of variables and UB is the upper bound of variables

4. Results and Discussion

Figures 2 and 3 show respectively a variation of Field Oil Efficiency and Field Water Production Rate with time for 48 simulation runs conducted using the input datasets presented in Table 3. The water breakthrough time, T_{bt} and recovery factor at breakthrough, RF_{bt} were obtained from simulation results at the onset of water production and are presented in Table 4.

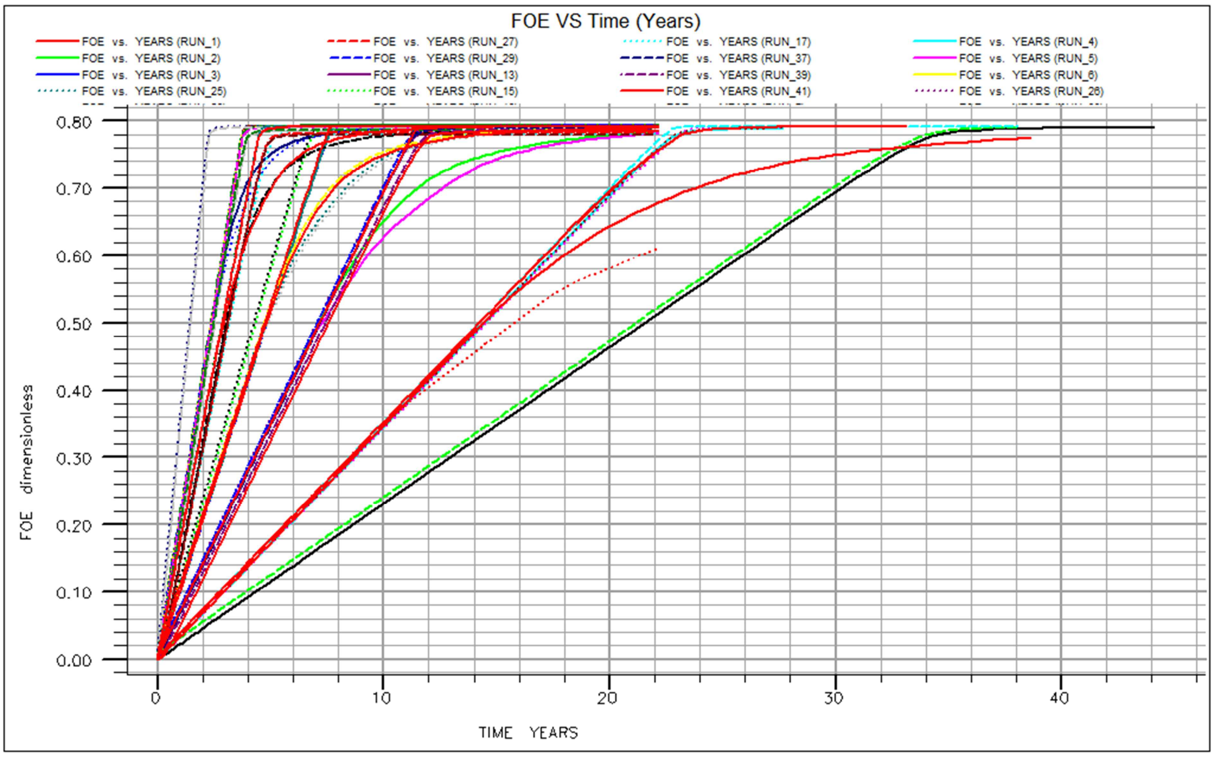

Figure 2. *Field Oil Efficiency versus Time for different simulation runs.*

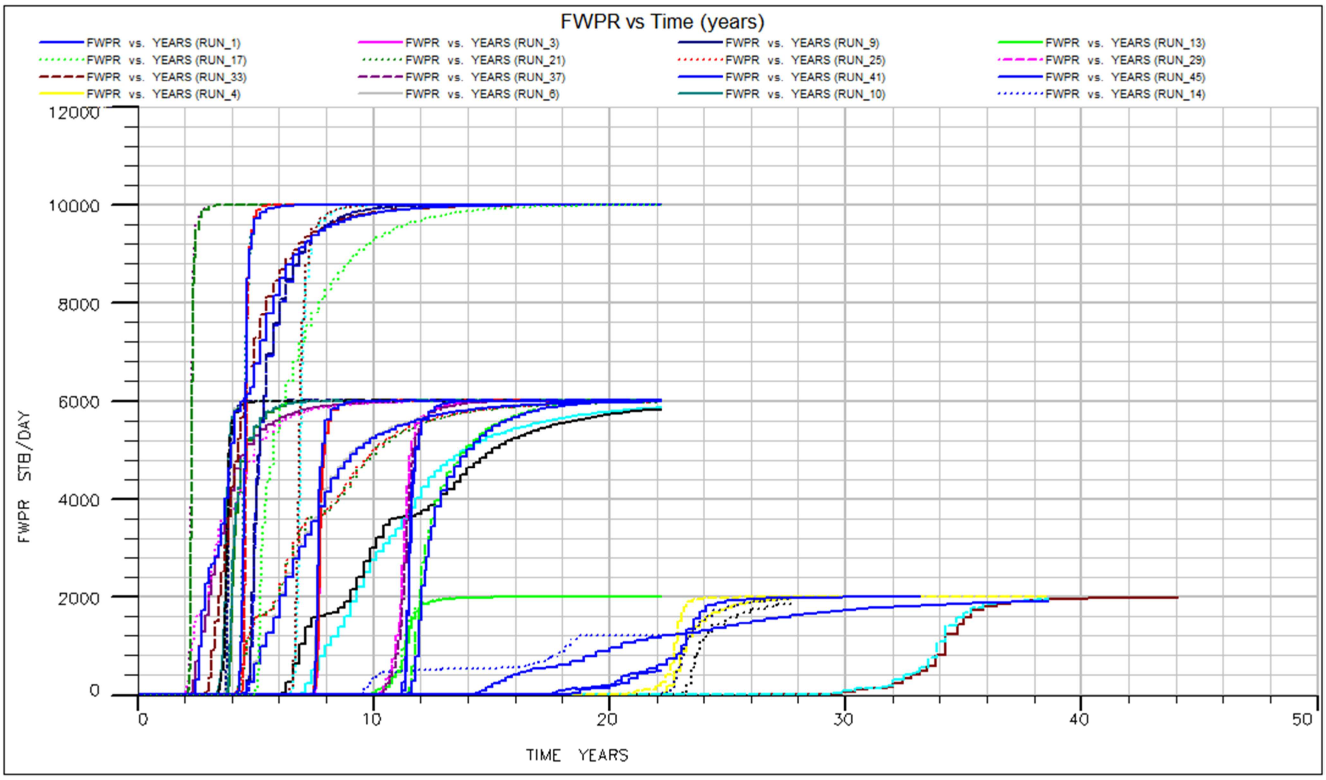

Figure 3. *Field Water Production Rate versus Time for different simulation runs.*

Table 4. *Time and Recovery Factor at water breakthrough for 48 input dataset.*

Run	Porosity %	Horizontal Permeability, md	Water viscosity cp	Bottom hole pressure, psi	Injection rate, STB/day	Vertical Permeability md	T_{bt}, Years	FOE, fraction
1	0.4	3230	5.5	3550	2000	2435	17.245	0.605
2	0.6	1715	1	3550	6000	75	6.847	0.48035
3	0.4	1715	10	3700	6000	2435	7.456	0.7679
4	0.4	200	5.5	3550	2000	75	22.17	0.76062
5	0.6	1715	1	3550	6000	2435	6.026	0.42285
6	0.4	1715	10	3400	6000	75	7.391	0.7714
7	0.6	1715	5.5	3700	2000	1255	29.29	0.67929
8	0.2	200	5.5	3700	6000	1255	3.778	0.74711
9	0.2	1715	10	3550	6000	2435	3.694	0.76755
10	0.2	3230	5.5	3400	6000	1255	3.286	0.69501
11	0.2	1715	1	3550	6000	75	2.19	0.4604
12	0.4	1715	10	3700	6000	75	7.392	0.76381
13	0.2	1715	5.5	3700	2000	1255	9.72	0.67634
14	0.4	3230	1	3550	2000	1255	9.029	0.31936
15	0.6	1715	5.5	3700	10000	1255	6.437	0.73959
16	0.2	1715	1	3550	6000	2435	1.917	0.40447
17	0.4	200	10	3550	10000	1255	5.009	0.76607
18	0.4	200	10	3550	2000	1255	23.08	0.77929
19	0.2	1715	10	3550	6000	75	3.699	0.76944
20	0.6	1715	5.5	3400	10000	1255	6.432	0.74762
21	0.4	1715	1	3700	6000	2435	4.106	0.42779
22	0.4	200	5.5	3550	2000	2435	22.17	0.76084
23	0.4	3230	5.5	3550	10000	75	4.212	0.73438
24	0.2	1715	5.5	3400	10000	1255	2.097	0.73166
25	0.4	1715	1	3400	6000	2435	4.102	0.43631
26	0.2	1715	5.5	3400	2000	1255	9.717	0.68488
27	0.4	200	5.5	3550	10000	2435	4.575	0.73638
28	0.6	1715	5.5	3400	2000	1255	29.292	0.68801
29	0.6	3230	5.5	3400	6000	1255	10.131	0.7127

Run	Porosity %	Horizontal Permeability, md	Water viscosity cp	Bottom hole pressure, psi	Injection rate, STB/day	Vertical Permeability md	T_{bt}, Years	FOE, fraction
30	0.4	3230	10	3550	2000	1255	20.698	0.72469
31	0.2	3230	5.5	3700	6000	1255	3.373	0.70324
32	0.4	1715	1	3400	6000	75	4.38	0.46619
33	0.4	200	1	3550	10000	1255	2.873	0.47574
34	0.2	1715	5.5	3700	10000	1255	2.151	0.74074
35	0.4	200	5.5	3550	10000	75	4.654	0.745
36	0.2	200	5.5	3400	6000	1255	3.742	0.74868
37	0.6	3230	5.5	3700	6000	1255	10.267	0.71359
38	0.4	3230	10	3550	10000	1255	4.441	0.76981
39	0.6	200	5.5	3400	6000	1255	11.361	0.75635
40	0.6	1715	10	3550	6000	2435	11.223	0.77613
41	0.4	3230	1	3550	10000	1255	2.329	0.40927
42	0.4	1715	10	3400	6000	2435	7.452	0.77741
43	0.4	200	1	3550	2000	1255	13.96	0.48528
44	0.4	3230	5.5	3550	10000	2435	4.207	0.7333
45	0.6	200	5.5	3700	6000	1255	11.363	0.74896
46	0.4	3230	5.5	3550	2000	75	18.338	0.64302
47	0.4	1715	1	3700	6000	75	4.382	0.45625
48	0.6	1715	10	3550	6000	75	11.089	0.76804

The 48 input datasets with their corresponding water breakthrough times, T_{bt} and recovery factors at breakthrough, RF_{bt} obtained from simulation and presented in Table 4 were entered into the Design Expert Software for further analysis and generation of the proxy model.

Table 5. *Comparison between different transformation functions for Water Breakthrough Time.*

Transformation Function	R^2	Adjusted R^2	Predicted R^2	Difference	Adeq Precision	Standard Deviation
None	0.9701	0.9609	0.9407	0.0202	39.4952	1.41
Square Root	0.9914	0.9881	0.9801	0.0080	63.7149	0.1179
Natural Log	0.9987	0.9982	0.9969	0.0013	165.5765	0.0315
Base 10 Log	0.9987	0.9982	0.9969	0.0013	165.5765	0.0137
Inverse Square Root	0.9940	0.9915	0.9859	0.0056	68.5338	0.0133
Inverse	0.9764	0.9691	0.9538	0.0153	41.3408	0.0224
Power	0.9701	0.9609	0.9407	0.0202	39.4952	1.41

Table 6. *Comparison between different transformation functions for Recovery Factor at Breakthrough.*

Transformation Function	R^2	Adjusted R^2	Predicted R^2	Difference	Adeq Precision	Standard Deviation
None	0.9946	0.9929	0.9891	0.0038	82.0438	0.0115
Square Root	0.9940	0.9920	0.9840	0.0080	74.9254	0.0081
Natural Log	0.9909	0.9878	0.9728	0.015	61.7610	0.0266
Base 10 Log	0.9919	0.9818	0.9574	0.0244	35.1110	0.0141
Inverse Square Root	0.9849	0.9797	0.9516	0.0281	48.8361	0.0232
Inverse	0.9752	0.9667	0.9178	0.0489	38.9646	0.0812
Power	0.9946	0.9929	0.9891	0.0038	82.0438	0.0115

4.1. Selection of Transformation Function

Different transformation functions with a modified quadratic regression model were evaluated to determine that which gives the highest R^2 value, the lowest difference between Adjusted R^2 and Predicted R^2, the highest value of Adeq Precision and the lowest standard deviation. Tables 5 and 6 show a comparison of different transformation functions for time and recovery factor at water breakthrough respectively.

Results from Table 5 shows that "Natural Log" and "Log_{10}" transformation functions both met the stated criteria in comparison with others while results from Table 6 shows that the "none" and "Power" transformation functions met the stated criteria. However, the "Log_{10}" resulted in a lower standard deviation for T_{bt} and was selected. For Recovery Factor at water breakthrough, the "none" transformation function was selected as both "None" and "Power" transformation functions resulted in the same standard deviation for the modified quadratic model.

The Box-Cox Plot for Power-law transformations was generated for each of water breakthrough time and recovery factor at breakthrough as shown in Figures 4 and 5 respectively. These plots gave recommendations of the best transformation function for each response using the modified quadratic model.

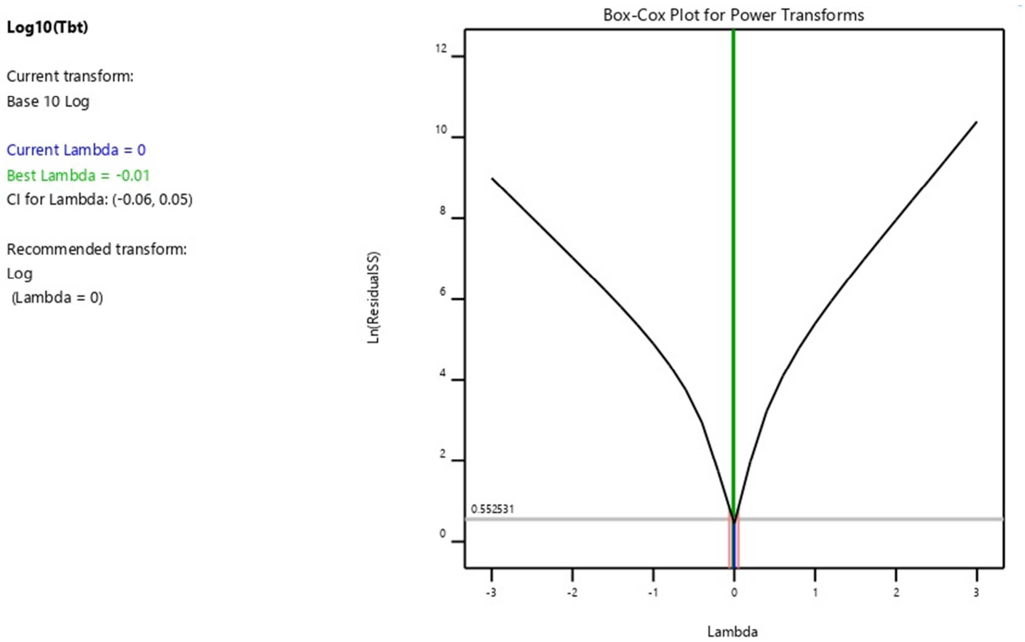

Figure 4. *Water Breakthrough Time Box-Cox Plot for Power Transformations.*

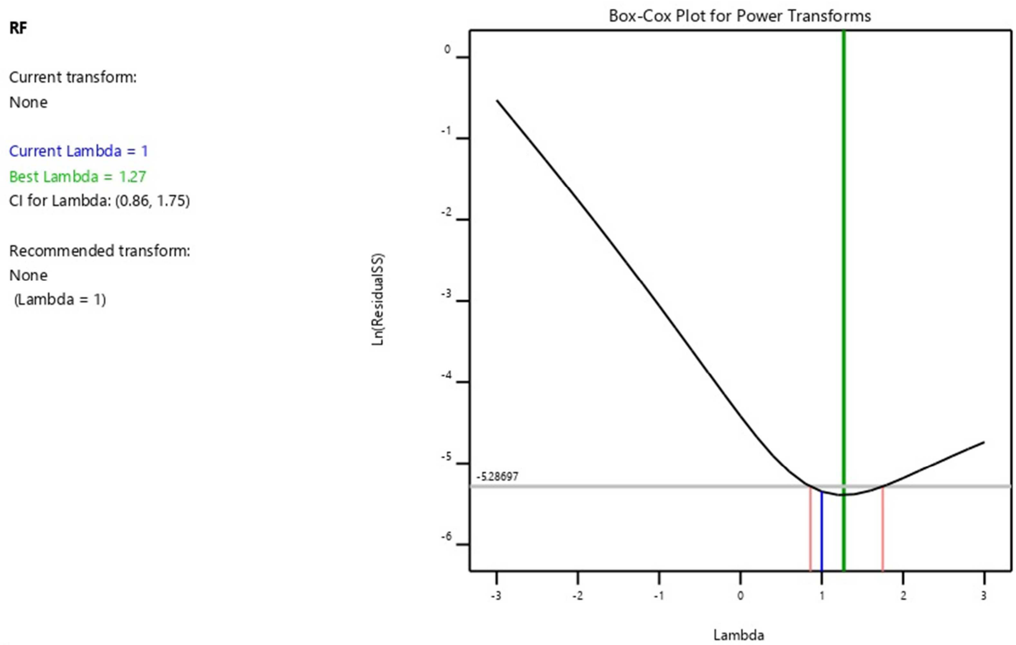

Figure 5. *Recovery Factor Box-Cox Plot for Power Transformations.*

From Figures 4 and 5, the recommended transformation functions were Log_{10} and None for T_{bt} and RF respectively which are in agreement with recommendations from Tables 5 and 6. It can be inferred here that the recommended transformation functions were accurate. Using the recommended transformation functions, proxy models for T_{bt} and RF were developed using the modified quadratic regression model. The proxy models for T_{bt} and RF_{bt} are given by equations 6 and 7 respectively.

$$Log_{10}(T_{bt}) = 0.613282 + 2.59821 * X1 + -5.43121e-05 * X2 + 0.0766922 * X3 + 9.38107e-06 * X4 + -0.000170686 * X5 + -1.85851e-05 * X6 + 3.31563e-06 * X2 * X3 + 2.113e-09 * X2 * X5 + 2.14181e-06 * X3 * X6 + -1.73581 * X1^2 + -0.00515332 * X3^2 + 7.09961e-09 * X5^2 \quad (6)$$

$$RF_{bt} = 0.45516 + 0.0216583 * X1 + -6.46865e-05 * X2 + 0.0940642 * X3 + -1.43361e-05 * X4 + 9.82496e-06 * X5 + -1.9342e-05 * X6 + 3.32838e-06 * X2 * X3 + 4.30621e-09 * X2 * X5 + 2.21363e-06 * X3 * X6 + -0.00599173 * X3^2 + -9.80234e-10 * X5^2 \quad (7)$$

4.2. Model Validation

The sets of input data in Table 4 were used in calculating T_{bt} and RF_{bt} using the developed proxy models shown in equations 6 and 7 respectively. The average absolute percentage error and root mean squared error between actual and predicted T_{bt} and RF_{bt} were calculated using equations 4 and 5 respectively. Table 7 shows the simulated, predicted, and percentage error values of each data point for T_{bt} and RF_{bt} respectively.

Table 7. *Comparison of Actual and Predicted values of time and recovery factor at water breakthrough.*

Run	X1 %	X2 Md	X3 cp	X4 psi	X5 STB/day	X6 md	Simulated T_{bt} Years	Predicted T_{bt}, Years	Percentage Error for T_{bt}, %	Simulated RF, fraction	Predicted FOE, fraction	Percentage Error for RF, %
1	0.4	3230	5.5	3550	2000	2435	17.245	17.44369	1.152156	0.605	0.62532	3.358691
2	0.6	1715	1	3550	6000	75	6.847	6.556599	4.24129	0.48035	0.466793	2.822374
3	0.4	1715	10	3700	6000	2435	7.456	7.673001	2.910419	0.7679	0.773169	0.686219
4	0.4	200	5.5	3550	2000	75	22.17	22.60499	1.962084	0.76062	0.756671	0.519143
5	0.6	1715	1	3550	6000	2435	6.026	5.996149	0.495362	0.42285	0.42637	0.832393
6	0.4	1715	10	3400	6000	75	7.391	7.506976	1.569156	0.7714	0.770876	0.06796
7	0.6	1715	5.5	3700	2000	1255	29.29	29.63525	1.178715	0.67929	0.693177	2.04433
8	0.2	200	5.5	3700	6000	1255	3.778	3.822703	1.183256	0.74711	0.753109	0.803019
9	0.2	1715	10	3550	6000	2435	3.694	3.734335	1.091908	0.76755	0.770988	0.447949
10	0.2	3230	5.5	3400	6000	1255	3.286	3.225967	1.826929	0.69501	0.695165	0.022233
11	0.2	1715	1	3550	6000	75	2.19	2.152057	1.732547	0.4604	0.458129	0.493178
12	0.4	1715	10	3700	6000	75	7.392	7.555781	2.215653	0.76381	0.766575	0.361992
13	0.2	1715	5.5	3700	2000	1255	9.72	9.727108	0.073126	0.67634	0.684514	1.208506
14	0.4	3230	1	3550	2000	1255	9.029	9.885521	9.486335	0.31936	0.324867	1.724356
15	0.6	1715	5.5	3700	10000	1255	6.437	6.559541	1.903697	0.73959	0.736755	0.383274
16	0.2	1715	1	3550	6000	2435	1.917	1.968102	2.665739	0.40447	0.417706	3.272543
17	0.4	200	10	3550	10000	1255	5.009	4.638531	7.396069	0.76607	0.760464	0.731775
18	0.4	200	10	3550	2000	1255	23.08	22.22926	3.686029	0.77929	0.769077	1.31056
19	0.2	1715	10	3550	6000	75	3.699	3.677286	0.587024	0.76944	0.764394	0.655843
20	0.6	1715	5.5	3400	10000	1255	6.432	6.517171	1.324179	0.74762	0.741056	0.877962
21	0.4	1715	1	3700	6000	2435	4.106	4.043893	1.512601	0.42779	0.419888	1.847238
22	0.4	200	5.5	3550	2000	2435	22.17	21.78433	1.739593	0.76084	0.739757	2.771005
23	0.4	3230	5.5	3550	10000	75	4.212	4.249844	0.898475	0.73438	0.738004	0.493473
24	0.2	1715	5.5	3400	10000	1255	2.097	2.139116	2.008391	0.73166	0.732393	0.100164
25	0.4	1715	1	3400	6000	2435	4.102	4.017772	2.053339	0.43631	0.424189	2.778178
26	0.2	1715	5.5	3400	2000	1255	9.717	9.664278	0.542576	0.68488	0.688814	0.574471
27	0.4	200	5.5	3550	10000	2435	4.575	4.545688	0.640702	0.73638	0.731144	0.711013
28	0.6	1715	5.5	3400	2000	1255	29.292	29.44382	0.518311	0.68801	0.697478	1.376108
29	0.6	3230	5.5	3400	6000	1255	10.131	9.828443	2.986445	0.7127	0.703828	1.244866
30	0.4	3230	10	3550	2000	1255	20.698	19.75276	4.566815	0.72469	0.700022	3.403884
31	0.2	3230	5.5	3700	6000	1255	3.373	3.24694	3.737327	0.70324	0.690864	1.759898
32	0.4	1715	1	3400	6000	75	4.38	4.393306	0.30379	0.46619	0.464611	0.338599
33	0.4	200	1	3550	10000	1255	2.873	2.858706	0.497537	0.47574	0.476074	0.070107
34	0.2	1715	5.5	3700	10000	1255	2.151	2.153023	0.094045	0.74074	0.728092	1.707478
35	0.4	200	5.5	3550	10000	75	4.654	4.716934	1.352246	0.745	0.748058	0.410529
36	0.2	200	5.5	3400	6000	1255	3.742	3.798012	1.496835	0.74868	0.75741	1.166088
37	0.6	3230	5.5	3700	6000	1255	10.267	9.89234	3.649163	0.71359	0.699527	1.970738
38	0.4	3230	10	3550	10000	1255	4.441	4.637696	4.429093	0.76981	0.795792	3.375128
39	0.6	200	5.5	3400	6000	1255	11.361	11.57127	1.850819	0.75635	0.766074	1.285593
40	0.6	1715	10	3550	6000	2435	11.223	11.37727	1.374593	0.77613	0.779652	0.453732
41	0.4	3230	1	3550	10000	1255	2.329	2.320994	0.343745	0.40927	0.420637	2.777283
42	0.4	1715	10	3400	6000	2435	7.452	7.623439	2.300576	0.77741	0.77747	0.007757
43	0.4	200	1	3550	2000	1255	13.96	13.6998	1.863927	0.48528	0.484686	0.122326
44	0.4	3230	5.5	3550	10000	2435	4.207	4.095556	2.649023	0.7333	0.72109	1.665108
45	0.6	200	5.5	3700	6000	1255	11.363	11.6465	2.494934	0.74896	0.761773	1.71074
46	0.4	3230	5.5	3550	2000	75	18.338	18.10083	1.293326	0.64302	0.642234	0.122192
47	0.4	1715	1	3700	6000	75	4.382	4.421868	0.909813	0.45625	0.460311	0.890006
48	0.6	1715	10	3550	6000	75	11.089	11.20346	1.032203	0.76804	0.773057	0.653222

A plot of simulated and predicted water breakthrough time and recover factor at breakthrough are shown in Figures 6 and 7 respectively.

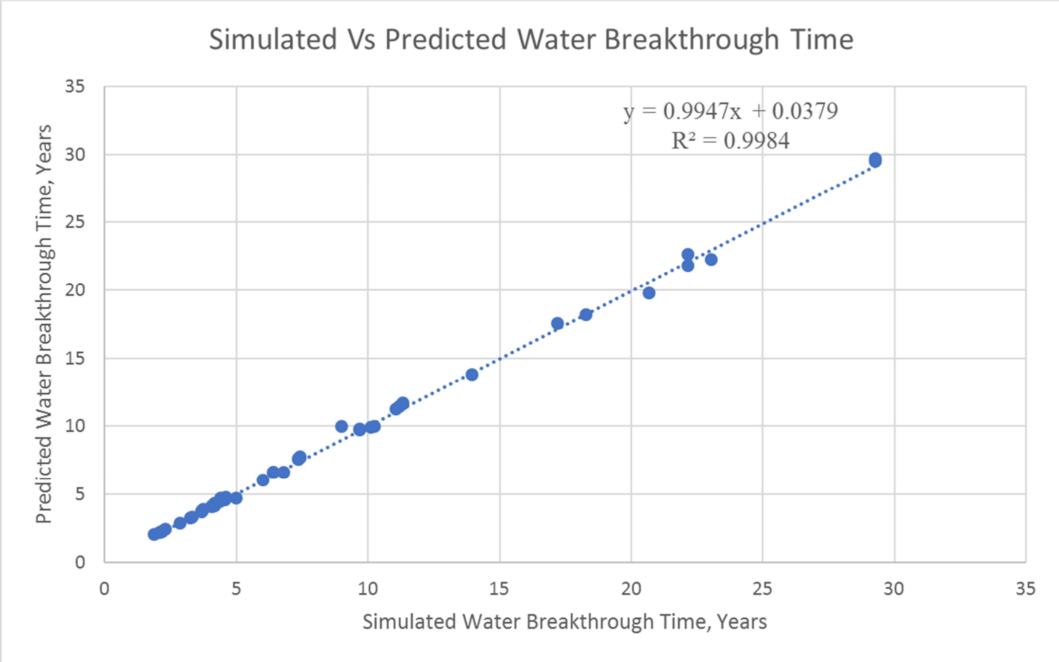

Figure 6. *Simulated versus Predicted Values for Water Breakthrough Time.*

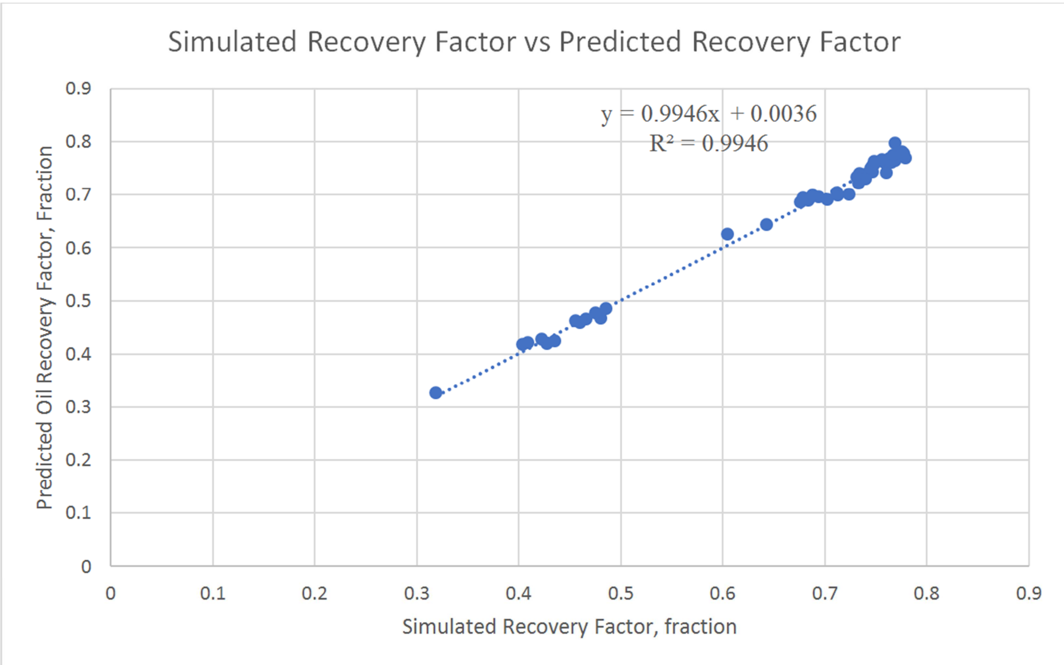

Figure 7. *Simulated versus Predicted Values for Oil Recovery Factor.*

Results from figures 6 and 7 show that the simulated and predicted values for each response are in close agreement with each other depicted by coefficients of determination R^2 of 0.9984 and 0.9946 for time and recovery factor at water breakthrough respectively which are relatively high. Also, the percentage errors of each data point for both cases were found to be less than 10%. The average absolute percentage error (AAPE) and root mean square error (RMSE) for time and recovery factor at water breakthrough were calculated using equations 5 and 6. The AAPE was found to be 2.038% and 1.217% while the RMSE was found to be 0.08 and 0.0000988 for time and recovery factor at water breakthrough respectively. This indicates that the developed models are accurate and reliable.

The developed proxy models were further validated by using input data sets within the design space that were not initially used in conducting any of the 48 simulation runs presented in Table 3. Six input data sets were used to run a simulation and calculate from the developed proxy models, T_{bt} and *RF* and their corresponding percentage errors which are presented in Table 8.

Table 8. *Percentage Error between Simulated and Predicted T_{bt} and RF for variables not used in model development.*

Run	X1 %	X2 Md	X3 cp	X4 psi	X5 STB/day	X6 md	Simulated T_{bt}, Years	Predicted T_{bt}, Years	Percentage Error,%	Simulated FOE, fraction	Predicted FOE, fraction	Percentage Error,%
1	0.39	298	5.7	3621	2867	2017	15.054	16.35461	7.952577	0.66102	0.7497	11.8398
2	0.42	3005	8.5	3456	7678	961	6.02	6.140251	1.958412	0.7682	0.7884	2.5726
3	0.52	1770	6.2	3648	7727	1095	7.249	7.498133	3.322595	0.74566	0.7591	1.7790
4	0.29	1601	5.2	3661	9144	484	3.37	3.276592	2.850768	0.73446	0.7273	0.9774
5	0.44	2812	7.3	3519	9254	688	5.199	5.437955	4.394209	0.7609	0.7848	3.0551
6	0.54	1641	9	3654	8350	1389	7.302	7.52961	3.022866	0.77481	0.7922	2.1989

Results from Table 8 shows that the actual and predicted values for T_{bt} and RF_{bt} were in agreement with each other depicted by percentage errors of less than 8% and 12%, and root mean square errors of 0.5606 and 0.0396 respectively. This shows that the developed models can be used to navigate the design space and in predicting T_{bt} and RF_{bt} for an oil reservoir.

4.3. Determination of Optimum Parameters to Maximize Time and Recovery Factor at Water Breakthrough Using Multi-Objective Genetic Algorithm

Studies have shown that optimizing the proxy model of a reservoir simulation model is computationally inexpensive and less time-consuming. Since two proxy models were developed in this study, a multi-objective genetic algorithm was used in determining the optimum sets of parameters of the developed proxy models that will delay water breakthrough time and maximize recovery factor at breakthrough.

Solutions of multi-objective genetic algorithms are illustrated using Pareto fronts. With the Pareto optimum set, the corresponding objective function values in the objective space are called the Pareto front. For a reservoir with porosity, horizontal and vertical permeability values fixed at the 0.2, 200 md, 75 md respectively, the optimization results are presented in Table 9 and illustrated by the Pareto front presented in Figure 8. The Pareto front shows a plot of objective function 1 (time at water breakthrough) versus objective function 2 (recovery factor at water breakthrough).

Results from figure 8 show the best value of the objective functions (T_{bt} and RF_{bt}) are 10.8978 years and 0.786 respectively. The corresponding optimum parameter values of viscosity of injected water (X3), bottom-hole pressure (X4), and water injection rate (X5) are 7.5278 cp, 3624.4 psi, and 2194.1 STB/day respectively.

The stated porosity, horizontal and vertical permeability values with the optimum parameter values were used to carry out a simulation study to check the validity of the optimization results. T_{bt} and RF_{bt} from simulation were found to be 10.27 years and 0.7647 respectively which are in agreement with the best objective function values obtained from optimization (10.8978 years and 0.786) as they resulted in percentage errors of 6.11% and 2.79% respectively.

Table 9. *Multi-Objective Optimization Results at fixed values of porosity (0.2), vertical (200 md) and horizontal (75 md) permeability*

S/N	T_{bt} *years*	RF_{bt} *fraction*	S/N	T_{bt}, *years*	RF_{bt} *fraction*	S/N	T_{bt}, *years*	RF_{bt} fraction	S/N	T_{bt} *years*	RF_{bt}, fraction
1	11.2381	0.7892	21	11.3427	0.78912	41	7.53653	0.79639	61	5.26707	0.80013
2	5.83497	0.79946	22	7.39181	0.79657	42	5.14451	0.80023	62	5.41244	0.79997
3	4.97401	0.80043	23	5.76303	0.79952	43	5.52397	0.79988	63	6.70429	0.79797
4	8.92592	0.79379	24	6.21597	0.79887	44	5.36046	0.80004	64	6.26782	0.79876
5	10.9858	0.7898	25	6.9818	0.79746	45	8.6246	0.7943	65	6.79282	0.79782
6	10.7096	0.7904	26	9.01284	0.79364	46	8.53491	0.79456	66	8.09376	0.79533
7	11.4207	0.78746	27	8.72101	0.79416	47	7.28383	0.7969	67	9.90748	0.7919
8	10.8517	0.79005	28	7.9726	0.79559	48	5.70659	0.79959	68	9.64578	0.79242
9	9.37997	0.79293	29	10.8198	0.79016	49	10.467	0.79079	69	10.9858	0.7898
10	6.41225	0.79849	30	5.22956	0.80021	50	10.5372	0.7907	70	8.35748	0.79485
11	4.89116	0.80049	31	6.10971	0.79898	51	9.08973	0.79343			
12	4.78715	0.80055	32	6.33229	0.79865	52	8.48294	0.79458			
13	6.93012	0.7976	33	6.63802	0.79808	53	7.16645	0.79701			
14	4.69686	0.80059	34	8.27655	0.79505	54	4.70604	0.80059			
15	8.17203	0.79522	35	7.5958	0.79635	55	10.665	0.7904			
16	10.3347	0.79099	36	6.51259	0.79836	56	7.70535	0.79614			
17	11.1037	0.78949	37	11.1002	0.78968	57	5.92903	0.7992			
18	9.50215	0.79251	38	7.85293	0.79586	58	7.35734	0.79681			
19	4.87309	0.80051	39	10.1699	0.79138	59	10.2263	0.79126			
20	11.1989	0.78946	40	9.74866	0.79199	60	7.90296	0.79579			

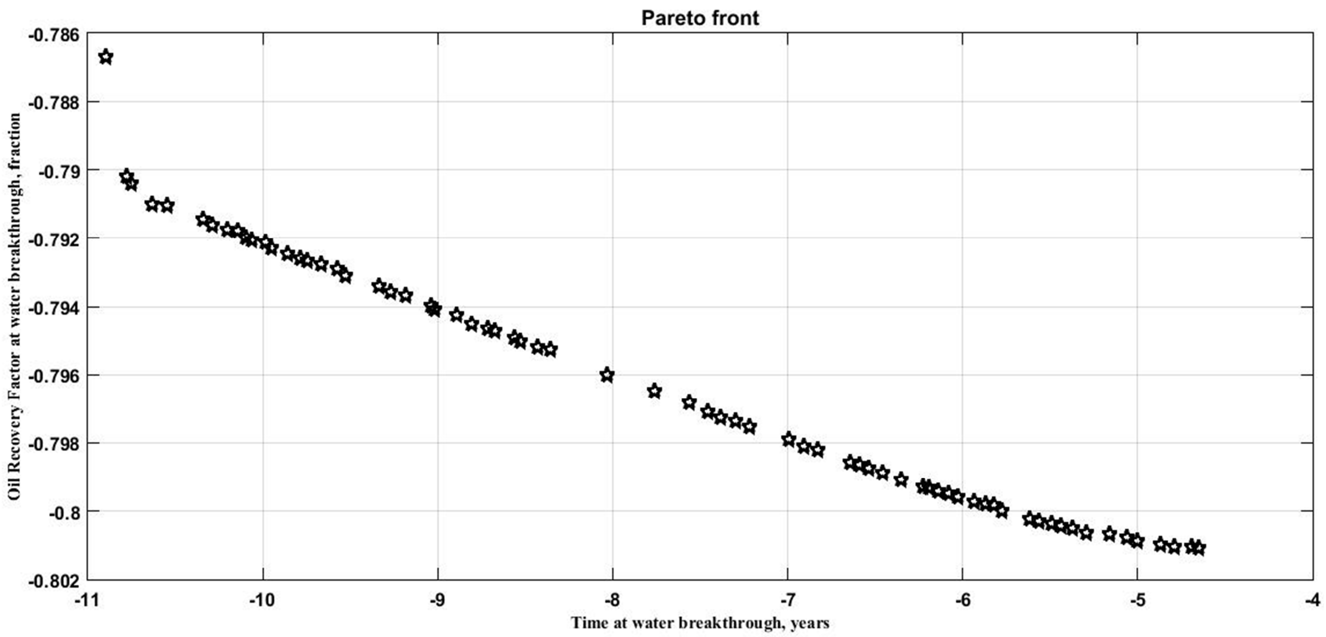

Figure 8. *Pareto Front for Time versus Recovery Factor at Water Breakthrough obtained using Multi-Objective Genetic Algorithm.*

5. Conclusion

A reservoir simulation proxy model for predicting time and recovery factor at water breakthrough using design of experiments and response surface methodology was developed. Development of the proxy models was based on data obtained from simulation and the transformation function which meets specific criteria or that recommended from the Box-Cox plot of power-law transformations.

In this study, the proxy models showed the relationship between water breakthrough time, T_{bt} and recovery factor at breakthrough, RF_{bt} with porosity, horizontal permeability, the viscosity of injected water, bottom-hole pressure of producer, water injection rate, and vertical permeability.

The models were validated by comparing simulated and predicted results for the two cases and agreed with each other. We also validated the proxy models by setting six sets of input variables that were not initially used during model development in simulating and predicting water breakthrough time and recovery factor at breakthrough. The simulated and predicted results were also in good agreement with each other since a minimal percentage error was observed for all cases. This shows that the developed proxy model can be used in navigating the design space and in predicting water breakthrough time and recovery factor at breakthrough for a reservoir of known porosity horizontal and vertical permeability.

This reservoir simulation proxy model can also be used to easily optimize the water injection process as this can aid in determining the optimum parameter values that will increase water breakthrough time and maximize recovery factor at water breakthrough in a reservoir of known average porosity, horizontal and vertical permeability. We applied in this study a multi-objective genetic algorithm in MATLAB to determine optimum parameters required to delay water breakthrough time and recovery factor at breakthrough. Optimization results were also in agreement with simulations results. Hence, the model can be used to predict water breakthrough time and recovery factor at breakthrough in a reservoir of known porosity, horizontal and vertical permeability.

6. Further Work

Further work in this study will consider a reservoir model that is multi-layered and highly heterogeneous with more wells. It will also consider well placement and well control parameters in modeling the time and recovery at water breakthrough.

Nomenclature

ANOVA	Analysis of Variance
R^2	Coefficient of Determination
FOE	Field Oil Efficiency
FWPR	Field Water Production Rate
T_{bt}	Water Breakthrough Time
RF_{bt}	Recovery Factor at breakthrough
X1	Porosity
X2	Horizontal Permeability
X3	Viscosity of injected water
X4	Bottom-hole pressure of production well
X5	Water Injection rate
X6	Vertical Permeability
Y_{sim}	Actual Values
Y_{pred}	Predicted Values
n	Number of Simulation runs
AAPE	Average Absolute Percentage Error
RMSE	Root Mean Square Error

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